

$$\begin{pmatrix} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{pmatrix} \xrightarrow{\pi_1 = \pi_1 / 2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & 4 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{pmatrix} \xrightarrow{\substack{\pi_2 = \pi_2 + 3\pi_1 \\ \pi_3 = \pi_3 + 2\pi_1}} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & 4 \\ 0 & +\frac{1}{2} & +\frac{1}{2} & 1 \\ 0 & 2 & 1 & 5 \end{pmatrix}$$

$$\rightarrow \pi_2 / \frac{1}{2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 1 & 5 \end{pmatrix} \rightarrow \pi_3 = \pi_3 - 2\pi_2 \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$C_{2,1} = C_{3,3}$

$$\pi_1 = \pi_1 + \frac{1}{2} \pi_3$$

$$\pi_2 = \pi_2 - \pi_3$$

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$C_{2,1} = C_{2,2}$

$$\pi_1 = \pi_1 - \frac{1}{2} \pi_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 3 & 3 & 9 \\ 1 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 3 & 3 & 9 \\ 1 & -1 & 1 \end{pmatrix} \xrightarrow{\substack{\pi_2 = \pi_2 - 3\pi_1 \\ \pi_3 = \pi_3 - \pi_1}} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & +\frac{3}{2} & \frac{21}{2} \\ 0 & -\frac{1}{2} & +\frac{1}{2} \end{pmatrix}$$

$$\pi_2 = \pi_2 / \frac{3}{2} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 7 \\ 0 & -\frac{1}{2} & +\frac{1}{2} \end{pmatrix} \xrightarrow{\pi_3 = \pi_3 + \frac{1}{2}\pi_2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 7 \\ 0 & 0 & \frac{9}{2} \end{pmatrix}$$

$$\rightarrow \pi_3 = \pi_3 / \frac{9}{2} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$C_{2,1} = 33$

$$\pi_1 = \pi_1 + \frac{1}{2} \pi_3$$

$$\pi_2 = \pi_2 - 7\pi_3$$

$C_{2,1} = 2, 2$

$$\pi_1 = \pi_1 - \frac{1}{2} \pi_3 =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 4 \\ 6 & 8 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 12 + 12 & 21 + 16 \\ 20 + 6 & 35 + 8 \end{pmatrix} = \begin{pmatrix} 24 & 37 \\ 26 & 43 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 5 & 7 \\ 3 & 6 & 8 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 9 \\ 4 & 2 \\ 3 & 5 \end{pmatrix}$$

$$C \cdot D = \begin{pmatrix} 2 + 20 + 21 & 18 + 10 + 35 \\ 3 + 24 + 24 & 27 + 12 + 40 \end{pmatrix} = \begin{pmatrix} 43 & 63 \\ 51 & 79 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{pmatrix} \xrightarrow{\kappa_1 = \kappa_1 / 2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$$

$$\begin{array}{l} \kappa_2 = \kappa_2 + 3\kappa_1 \\ \kappa_3 = \kappa_3 + 2\kappa_1 \end{array} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 1 \end{pmatrix} \xrightarrow{\kappa_2 = \kappa_2 / \frac{1}{2}} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\kappa_3 = \kappa_3 - 2\kappa_2 \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \xrightarrow{\kappa_3 = \kappa_3 / -1} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_{L,K} = 3, 3$$

$$\begin{array}{l} \kappa_1 = \kappa_1 + \frac{1}{2}\kappa_3 \\ \kappa_2 = \kappa_2 - \kappa_3 \end{array} \rightarrow \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad C_{L,K} = 2, 2$$

$$\kappa_1 = \kappa_1 - \frac{1}{2}\kappa_2 \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 & 5 & -1 \\ 1 & 3 & 9 \\ 3 & 6 & 2 \end{pmatrix} \xrightarrow{\kappa_1 = \kappa_1 / 2} \begin{pmatrix} 1 & \frac{5}{2} & -\frac{1}{2} \\ 1 & 3 & 9 \\ 3 & 6 & 2 \end{pmatrix}$$

$$\begin{array}{l} \kappa_2 = \kappa_2 - \kappa_1 \\ \kappa_3 = \kappa_3 - 3\kappa_1 \end{array} \rightarrow \begin{pmatrix} 1 & \frac{5}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{9}{2} \\ 0 & -\frac{3}{2} & \frac{5}{2} \end{pmatrix} \xrightarrow{\kappa_2 = \kappa_2 / \frac{1}{2}} \begin{pmatrix} 1 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 9 \\ 0 & -\frac{3}{2} & \frac{5}{2} \end{pmatrix}$$

$$\kappa_3 = \kappa_3 + \frac{3}{2}\kappa_2 \rightarrow \begin{pmatrix} 1 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 9 \\ 0 & 0 & 16 \end{pmatrix} \xrightarrow{\kappa_3 = \kappa_3 / 16} \begin{pmatrix} 1 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{L,K} = 3, 3$$

$$\begin{aligned} \pi_1 &= \pi_1 + \frac{1}{2} \pi_3 \\ \pi_2 &= \pi_2 - \frac{1}{9} \pi_3 \end{aligned} \quad \begin{pmatrix} 1 & \frac{5}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{L,K} = 2, 2 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\pi_1 = \pi_1 - \frac{5}{2} \pi_2$$

$$A = \begin{pmatrix} 3 & -2 \\ 5 & 4 \\ 1 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 0 & 5 \\ -3 & 9 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{array}{ccc|ccc} 6+12 & 0-8 & 15-2 & 18 & -8 & 13 \\ 10-24 & 0+28 & 25+4 & -14 & 28 & 32 \\ 2-12 & 0+16 & 5+4 & -10 & +16 & +9 \end{array}$$

$$A = \begin{pmatrix} 1 & -5 & 3 \\ 9 & 2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 \\ -1 & 4 \\ 5 & 6 \end{pmatrix}$$

$$\begin{array}{ccc|cc} 2+5+16 & 3-20+18 & & 22 & 1 \\ 8-2+0 & 12+8+0 & & 6 & 20 \end{array}$$